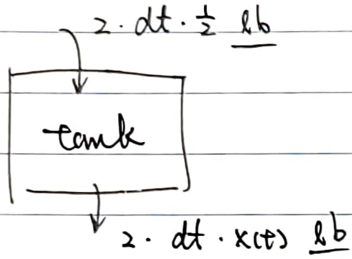
Pro. 3. Sol:

We may assume that the poured solution will mixed into the original solution immediately, i.e. the concentration is constant in the tank for a certain instant, which is denoted by $x(t)$ lb/gal.

In stage 1 (salt water poured in), consider the salt, in a tiny time period $dt$ min:



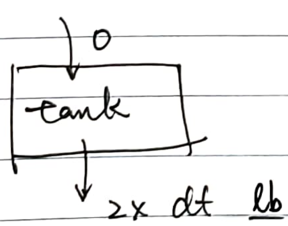

the change of salt in the tank is:

$$100\, dx(t) = \left(1 - 2x(t)\right) dt$$

i.e.
$$\begin{cases} \dfrac{d(100x)}{dt} = 1 - 2x, & 0 < t < 10 \\ x(0) = 0 \end{cases}$$

$$\Rightarrow \quad x(t) = \frac{1}{2}\left(1 - e^{-\frac{t}{50}}\right), \quad 0 \le t \le 10.$$

In stage 2:




$$100\, dx = -2x\, dt$$

i.e.
$$\begin{cases} \dfrac{dx}{dt} = \dfrac{-x}{50} & 10 < t < 20 \\ x(10) = \frac{1}{2}\left(1 - e^{-\frac{10}{50}}\right) \end{cases}$$

$$\Rightarrow \quad x(t) = \frac{1}{2}\left(e^{\frac{1}{5}} - 1\right) e^{-\frac{t}{50}}, \quad 10 \le t \le 20$$

i.e. $100x(20) = 50 e^{-\frac{2}{5}}\left(e^{\frac{1}{5}} - 1\right)$

i.e. the salt amount is $50 e^{-\frac{2}{5}}\left(e^{\frac{1}{5}} - 1\right)$ lb.

27. Sol:

a> in the steady state, by the Newton's law,

$$W = R + B$$

$$\Rightarrow \frac{4}{3}\pi a^3 [\rho - \rho']g = 6\pi\mu a|v|$$

i.e. $v_L = \frac{2a^2 g}{9\mu}\cdot(\rho - \rho')$, where we take "$\downarrow$" to be the positive direction.

b> As the droplet is held stationary,

$$\frac{4}{3}\pi a^3 g[\rho - \rho'] = Ee$$

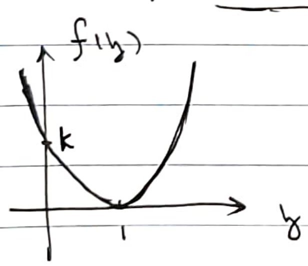
$$\Rightarrow e = \frac{4\pi a^3 g}{3E}(\rho - \rho')$$

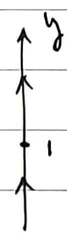
P88. 7. Sol:

a>


by the phase line analysis, we see that $y = 1$ is the only critical pt, the corresponding equilibrium is $\varphi(t) = 1$.

b>




indeed, $y(t) = \begin{cases} 1 - \frac{1}{C + kt}, & y_0 \neq 1 \\ 1, & y_0 = 1. \end{cases}$

i.e. $y = 1$ is the only critical pt.

c> refer to <a>, we get

$$y(t) = \frac{y_0 + (1 - y_0)kt}{1 + (1 - y_0)kt}.$$

15. Sol:

a> $$\frac{dy}{dt} = ry - \frac{r}{K}y^2$$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{y}\right) = \frac{r}{K} - r\cdot\left(\frac{1}{y}\right)$$

$$\Rightarrow y(t) = e^{rt}\left[\frac{1}{y_0} + \frac{e^{rt}-1}{K}\right]^{-1} \quad \text{or} \quad y \equiv 0.$$

Since $y_0 = \frac{K}{3}$, $\quad y(t) = e^{rt}\left(\frac{2+e^{rt}}{K}\right)^{-1}$

set $y(t) = 2y_0 = \frac{2}{3}K$, we get $t = \frac{1}{r}\log 4$; $\quad$ 55.452 yr

b> set $y_0 = \alpha K$, $y(T) = \beta K$,

we get $\beta K = e^{rT}\left(\frac{\frac{1}{\alpha} - 1 + e^{rT}}{K}\right)^{-1}$

$$\Rightarrow \frac{\beta}{\alpha} - \beta = (1-\beta)e^{rT}$$

$$\Rightarrow T = \frac{1}{r}\log\frac{\beta(1-\alpha)}{\alpha(1-\beta)} \quad ; \quad 175.78 \text{ yr}$$

17. Sol:

a> $$\frac{dy}{dt} = ry(\log K - \log y)$$

$$\Rightarrow \frac{d}{dt}(\log y) = r\log K - r\log y$$

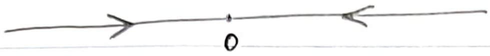
$$\Rightarrow y(t) = K\exp\left\{e^{-rt}\log\frac{y_0}{K}\right\}.$$

b> $y(2) \cong 0.7153\,K \cong 57.6 \times 10^6$ kg

c> $\tau \cong 2.215$ yr

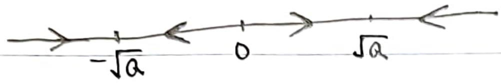
26. Sol:

a>




$(a<0)$


$(a=0)$

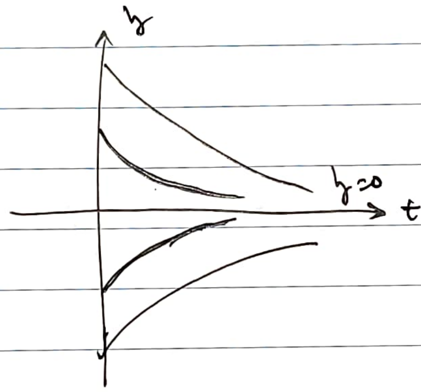



$(a>0)$.

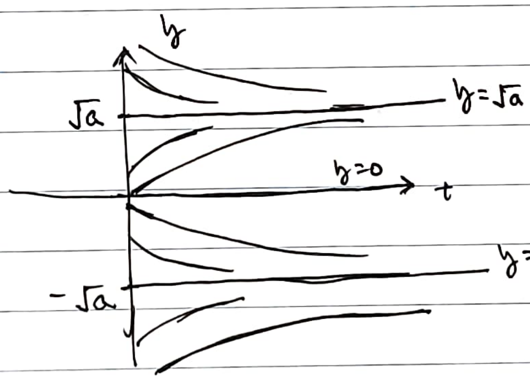
for $a \le 0$, $y=0$ is asymptotically stable

$a>0$, $y=0$ is unstable ; $y=\sqrt{a}$ & $y=-\sqrt{a}$ are asymptotically stable

b> for $a \le 0$.




for $a>0$




c>